The old 'problem' of supposed loss of energy when discharging one capacitor into another

Unusually, I find myself in agreement with Ivor on the old 'paradox' of the loss of energy when a charged capacitor is connected in parallel to a discharged capacitor via a switch.

This is an example of what I think I knew about as a teenager (though for sure I would not have had a good understanding of it then). It was also something for which practical experiments could be done on a kitchen table (provided no adults were around). Taking a 1950s-era electrolytic capacitor charged to 500V and carefully connecting it by a short piece of wire to a similar but discharged capacitor, there would be a large and noisy spark and easy to conclude (more or less correctly) that the 'missing energy' was used making the spark. The short piece of wire might also become too hot to hold, illustrating more energy being used up.

Moving on to my time in the army after leaving school, we certainly knew there of this 'paradox' and a few who had joined the army from a university engineering degree or similar were able to show by straightforward solution of differential equations that the resistance of the switch and any wires leading to it always generates exactly the correct amount of heat to account for the change in energy. This is true regardless of the resistor value so long as it is greater than zero and less than infinity. In the case of infinity, the answer is easy: the capacitor never discharges and there is no loss of energy. In the case of zero (of course assuming no inductance in the connections), infinite current flows for zero time and that leads to an indeterminate result best left to philosophers to debate, but which does not rule out the possibility of zero multiplied by infinity being equal to the lost energy.

The analysis which we could do in those days assumed a linear resistor – the result is just the same for any positive non-linear resistor, all that changes is that the differential equation to solve is non-linear and may have no analytic solution anyway, but standard methods of approximating the solution would yield the expected result. Maybe also valid for a time-varying resistor, but I am prepared to believe that I might be wrong about that.

If we propose an ideal switch, zero resistance wire, but some inductance along the wire then the configuration becomes a lossless series resonant circuit at the moment the switch is closed – the result is a constant amplitude sinusoid (of frequency easily calculated from the capacitor and inductor values). No energy is then lost, it simply goes from energy in the capacitor to energy in the inductor, to and fro forever, in the normal manner of resonance in a lossless resonator. So no 'missing energy' has to be accounted for.

For a more realistic case of a switch and wire that has some series resistance and some series inductance then after the closure of the switch, a damped sinusoid is generated, and of course that may involve a long or short decay depending on the values assigned. More complicated mathematically but still 'standard well-known stuff'.

I know that this example continues to crop up from time to time as a useful topic in the teaching of traditional circuit theory courses, and I certainly made use of it while teaching undergraduate electrical engineers.

What may be of interest is the dual but mathematically equivalent problem using two inductors. I knew of this just after I graduated and went to work for GEC in Coventry: the relay-design group used this to make fast-operating relays from standard Post Office relays. There are several ways to make a relay operate more quickly than normal: one is to use a far higher voltage so that the exponential current transient heads to a far higher value and so gets to the operating value more quickly (not a good solution because the high voltage may not be available in telephone exchange type equipment, and there is also a safety risk – even in the early 1960s the GEC had some concerns about safety of what it made). A second way is to remove most or all of the iron core of the relay – which reduces the inductance and so the current can increase much more quickly – but then the magnetic attraction to the armature is far less, so not easy to make a good solution that way. The answer is then to use the dual of the 'two capacitor in parallel connected by closing a switch'.

That means to have two ideal inductors in series (for simplicity let's say they are equal in value) and one has a closed switch across its terminals and a large circulating current flowing (other parts of the system arrange for getting the current there in the first place – just as in the two capacitor version, we do not spend time discussing how the charged capacitor came to be charged in the first place).

Energy is stored in the magnetic field produced by the circulating current, just as in the capacitor case it is stored in the electric field produced by the voltage across the charged capacitor. Since the inductor is ideal, the current would continue flowing for ever, just as in the case of the ideal charged capacitor, the voltage remains constant for ever.

When the closed switch is opened, the final stored energy in the two inductors is less than the initial stored energy in the inductor with the switch across it, so the same question arises: where does the missing energy to go to.

In the inductor case, the closed switch is opened, and that causes the current in the second inductor to rise to a final value very quickly, far quicker than if a normal voltage is applied to it to make it operate in the conventional way. Of course the relay does not really operate in zero time, any more than the charged capacitor really discharges in zero time. But it does give a practically-useful result (at least for the relay based systems designed in the early 1960s).

None of the above requires anything more than circuit theory, it is not necessary to get involved in field theory.

A capacitor can of course be described as a transmission line, and in some contexts that is useful, but not needed to describe and explain the 'two capacitor and switch 'paradox'.

I am not sure if there is a useful comparable way to describe an inductor as something 'distributed' – but that is a question for others and not important for the circuit theory based explanations.

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